



A BWXT/Bechtel Enterprise

**Y-12  
NATIONAL  
SECURITY  
COMPLEX**

MANAGED BY  
BWXT-Y12, L.L.C.  
FOR THE UNITED STATES  
DEPARTMENT OF ENERGY

Document No. Y/DK-2181

**Inherent Negative Biases in the Generalized  
Geometry Holdup (GGH) Model**

R. B. Oberer  
C. A. Gunn  
L. G. Chiang

May 16, 2006

Prepared by the  
Y-12 National Security Complex  
P.O. Box 2009  
Oak Ridge, TN 37831-8169  
managed by  
BWXT-Y12, L.L.C.  
for the  
U.S. DEPARTMENT OF ENERGY  
under contract DE-AC05-00OR22800

## DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

## COPYRIGHT NOTICE

The submitted manuscript has been authored by a subcontractor of the U. S. Government under contract DE-AC05-00OR-22800. Accordingly, the U. S. Government retains a paid-up, nonexclusive, irrevocable, worldwide license to publish or reproduce the published form of this contribution, prepare derivative works, distribute copies to the public, and perform publicly and display publicly, or allow others to do so, for U. S. Government purposes.

# Inherent Negative Biases in the Generalized Geometry Holdup Model

R. B. Oberer      C. A. Gunn      L. G. Chiang

September 7, 2006

## **Abstract**

Uranium which accumulates in process equipment and associated plumbing, ductwork, and filters is measured at Y-12 with portable NaI detectors using the Generalized Geometry Holdup (GGH) model. In this model all accumulations are modeled as points, lines, and areas. The width of point and line sources is also considered to account for detector response and the effects of self attenuation. The measurements are not precise but in theory the uncertainty will be reduced by a large number of unbiased measurements. However, the simplifications in the GGH model of real geometries result in measurement biases which violate this assumption. These biases need to be understood and corrected when they are significant. This paper reviews many of the biases inherent in the GGH model.

# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Generalized Geometry Holdup (GGH) Model</b>	<b>3</b>
2.1	Point . . . . .	5
2.2	Line . . . . .	5
2.3	Area . . . . .	6
2.4	Volume . . . . .	6
<b>3</b>	<b>Detector Efficiency</b>	<b>9</b>
<b>4</b>	<b>Attenuation Corrections</b>	<b>10</b>
<b>5</b>	<b>Background Subtraction</b>	<b>11</b>
<b>6</b>	<b>Conclusion</b>	<b>11</b>

## 1 Introduction

Holdup is measured and quantified at present by a method developed during the 1970s<sup>1</sup> called the generalized geometry holdup (GGH) method.<sup>2</sup> The original version of GGH reduced all geometries of holdup to three basic geometries: points, lines, and areas.<sup>3</sup> These original geometries were rather literal, in that a point was either dimensionless or the size of the calibration standard at the calibration distance, and a line was the same width. It was later realized that these literal definitions were unrealistic, and further were not useful for

---

<sup>1</sup>The earliest report of the method is found in R. H. Auguston and T. D. Reilly, *Fundamentals of Passive Nondestructive Assay of Fissionable Material*, Los Alamos Technical Report La-5651-M, September 1974, pp. 45-48.

<sup>2</sup>The most current description of the method can be found in P. A. Russo, *Gamma-Ray Measurements of Holdup Plant-Wide: Application Guide for Portable, Generalized Approach*, Los Alamos Technical Report LA-14206, June 2005.

<sup>3</sup>J. K. Sprinkle, Jr., R. Cole, M. L. Collins, S. Hsue, P. A. Russo, R. Siebelist, H. A. Smith, Jr., R. N. Ceo, and S. E. Smith, *Low-Resolution Gamma-Ray Measurements of Process Holdup*, Los Alamos Technical Report LA-UR-96-3482, October 1996.

estimating self attenuation. To overcome these limitations, a finite source correction was proposed for point and line sources.<sup>4</sup> The detector response to point and line sources was corrected for the fact that points and lines have a width and therefore extended beyond the center of the detector field-of-view by half of the width.

The width parameter also made an improved self-attenuation correction possible. As a consequence of the width, the result of point and line measurements can be converted into a density thickness<sup>5</sup> of  $^{235}\text{U}$  by dividing the grams of a point measurement by an area proportional to the square of the width and dividing the grams per distance from a line measurement by the width of the line.

## 2 Generalized Geometry Holdup (GGH) Model

To fully understand a holdup measurement, the exact GGH models and the implication of these models need to be understood.<sup>6</sup> The GGH calibration for a NaI detector is based on the symmetry of the detector. Both the detector crystal and collimator are cylindrical. Therefore, the relative detection efficiency for the detector field-of-view can be described by its radial response. A typical radial response is shown in Figure 1. The points of the radial response in the detector field-of-view, looking down the axis of the detector, are shown in Figure 2. The concentric circles in the field-of-view represent contours of equal detection efficiency. The relative detection efficiency between contour lines is defined by the radial response. A distribution of holdup is superimposed on the detector field-of-view to determine the average detection efficiency for that geometry. In GGH this distribution is reduced to one of three basic geometries: points, lines, and areas.

---

<sup>4</sup>P. A. Russo, T. R. Wenz, S. E. Smith, J. F. Harris, *Achieving Higher Accuracy in the Gamma-Ray Spectroscopic Assay of Holdup*, LA-13699-MS, September 2000,

<sup>5</sup>The density thickness was also often referred to as an areal density.

<sup>6</sup>Originally presented at the Fall 2004 Nuclear Nonproliferation Conference in Oak Ridge, Tennessee, November 17-18, 2004.

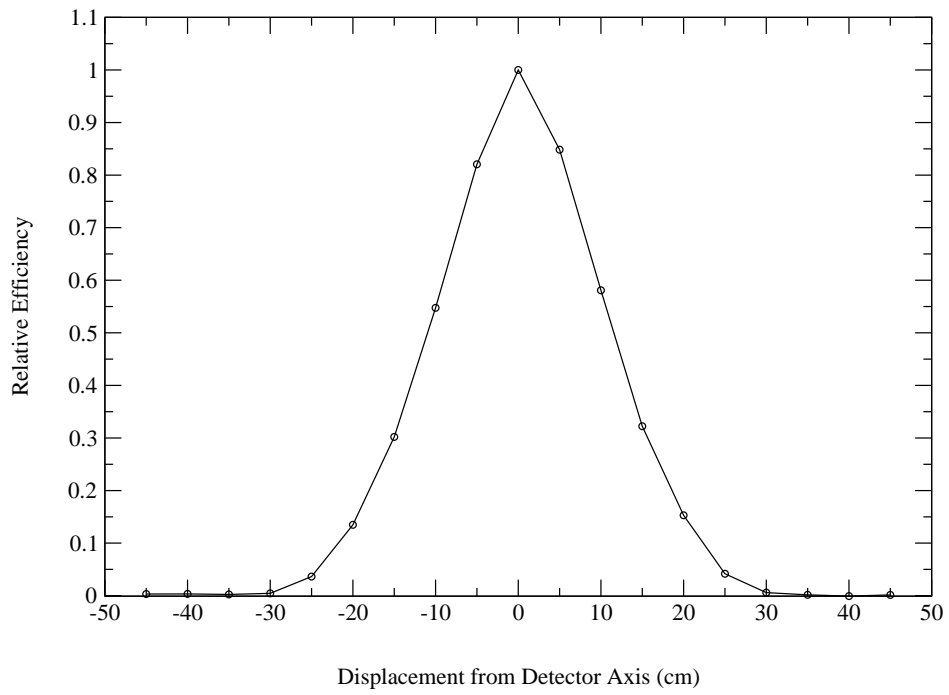


Figure 1: Radial response of a typical NaI detector.

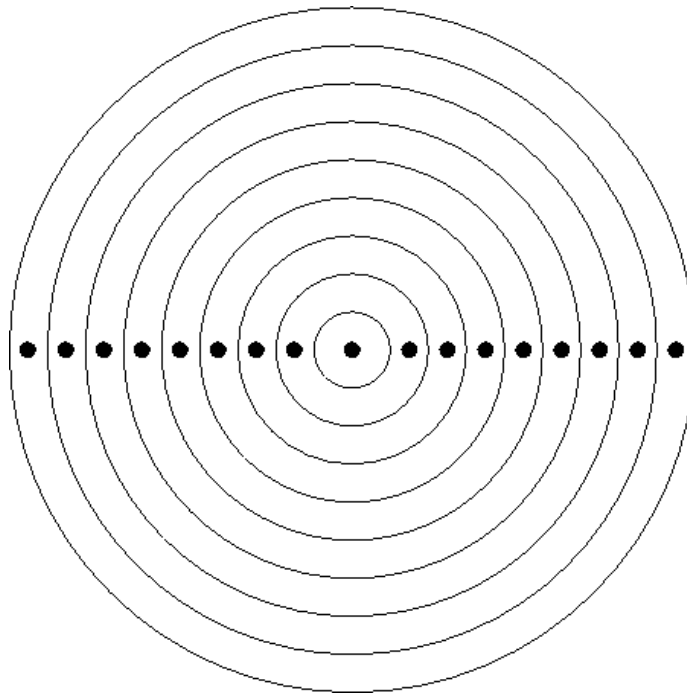


Figure 2: The points of a radial response, looking down the axis of a detector, showing contours of equal detection efficiency.

## 2.1 Point

A point in the GGH model, shown in Figure 3, is uniformly distributed within a circle centered in the detector field-of-view.<sup>7</sup> The width of the point is less than the width of the detector field-of-view. Once a point fills the detector field-of-view, it becomes an area. The plane of the circle is perpendicular to the axis of the detector. The detector response to a point source is inversely proportional to the square of the distance between the source and detector.

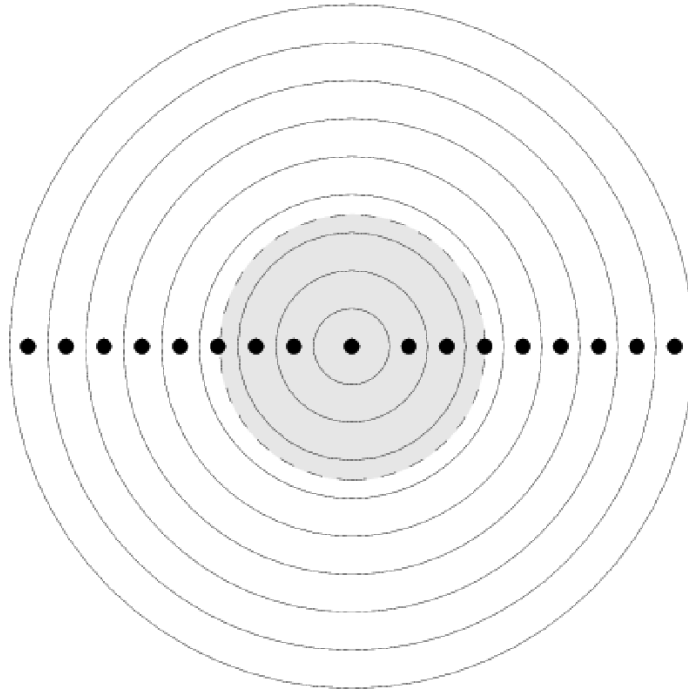


Figure 3: A point distribution superimposed on the detector field-of-view.

## 2.2 Line

A line in GGH, shown in Figure 4, intersects the center of the detector field-of-view, filling the field-of-view in one direction and has a width which is less than the width of the detector field-of-view in the opposite direction. Similarly if the width is as great or greater than that of the field-of-view, the line becomes an area. The line is perpendicular to the axis of the detector and the plane of the area is normal to that axis. The detector response to a line

---

<sup>7</sup>An alternative concept of a point is a rectangle with a width and height. J. R. Ewalt and B. D. Keele, Plutonium Finishing Plant Analytical Laboratory, *PFP Generalized Geometry Holdup Calculations and Total Measurement Uncertainty*, HNF-23383 Rev. 1, December 2004, Fluor Hanford, Inc.

source is inversely proportional to the distance between the source and detector as long as the line continues to fill the detector field-of-view in one direction.

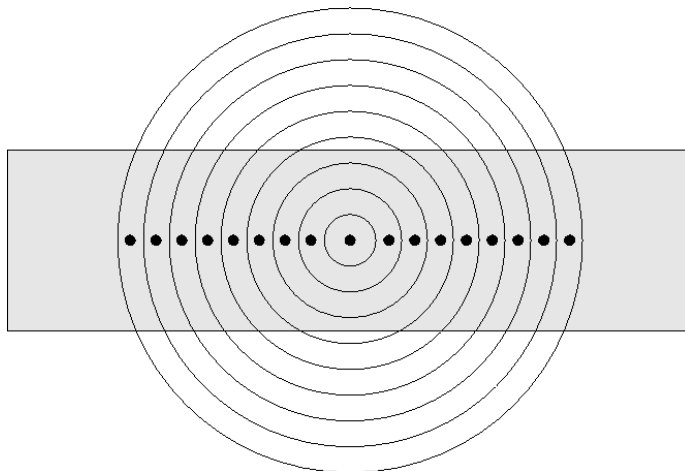


Figure 4: A line source superimposed on the detector field-of-view.

## 2.3 Area

An area, shown in Figure 5, is uniformly distributed and fills the detector field-of-view. The plane of the area is normal to the detector axis. The detector response to an area source is independent of distance between the source and detector as long as the area continues to fill the detector field-of-view.

## 2.4 Volume

In traditional GGH, the thickness of the holdup distribution is ignored. In reality every deposit has a thickness. In some cases such as filters or solutions the depth is considerable. In the case of a slab geometry as shown in Figure 6, the thickness can be ignored as long as the detector field-of-view remains filled. Once the detector field-of-view begins to extend beyond the volume as in Figure 7, a correction needs to be made otherwise a negative bias results. The correction becomes complicated by the fact that not all of the volume contributes equally because of self attenuation.<sup>8</sup>

---

<sup>8</sup>R. B. Oberer, N. B. Harold, C. A. Gunn, M. Brummett, L. G. Chiang, *Solution to High Self-Attenuation Corrections in HEPA Filter Measurements*, Y/DK-2152, Rev 1., October 2005



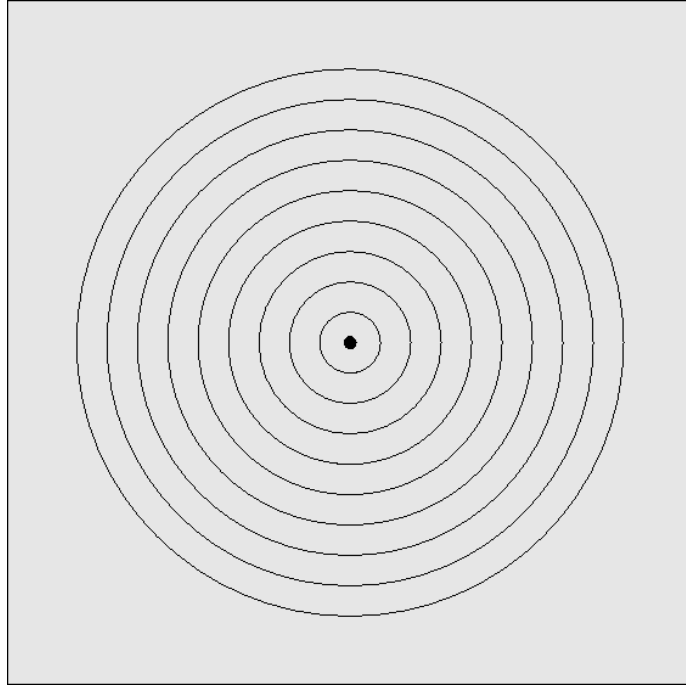


Figure 5: An area source superimposed on the detector field-of-view.

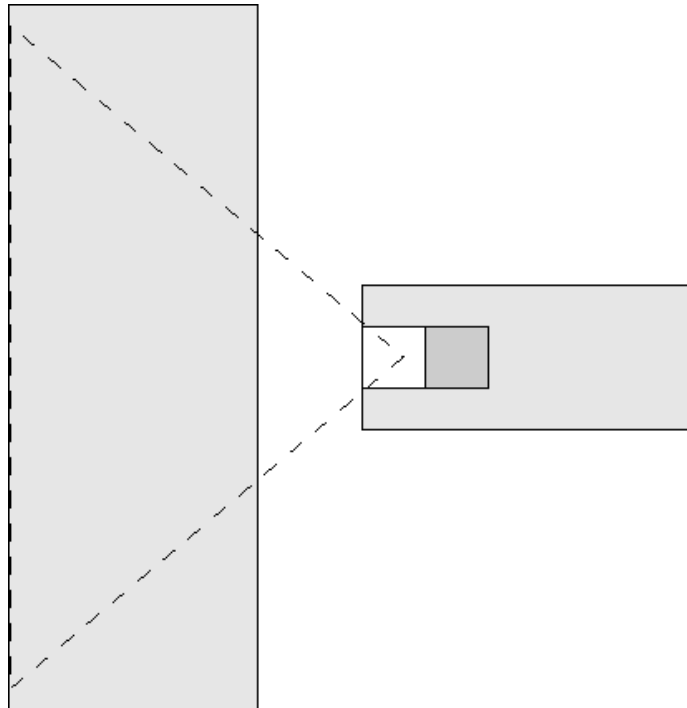


Figure 6: Volume fills the detector field-of-view.

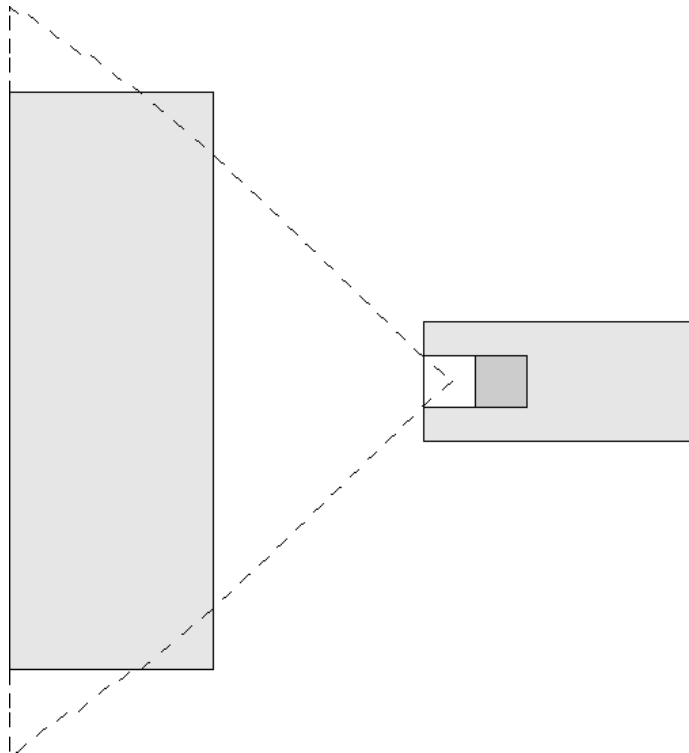


Figure 7: Detector field-of-view extends beyond volume.

### 3 Detector Efficiency

One purpose of these geometries is to estimate the average detector efficiency. In the traditional Los Alamos method a geometry correction factor is calculated for point and line distributions. This method estimated the average detector response to be half way between the response at the width of a line and the full response, and the square of this half-way estimate for a point. Although this simplistic approach works relatively well for narrow point and line configurations, it under corrects for wider geometries resulting in a negative bias in the measurement. A more sophisticated method for calculating the average detector response was developed at Y-12.<sup>9</sup>

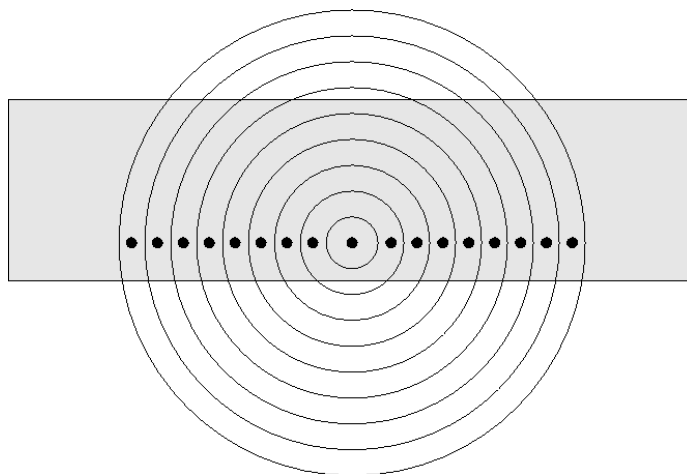


Figure 8: An example of a line source which is not centered within the detector field-of-view.

Any deviation from the centering requirement, as shown in Figure 8, reduces the average efficiency of the detector. As a point or line configuration is moved out of the center of the field-of-view, the count rate of the detector goes down.<sup>10</sup> If the aiming during calibration is more accurate than aiming during measurement, as it typically is, a negative bias is introduced by the centering requirement for point and line sources. A remedy for this bias is using a width for the average detector response that includes this jitter in the detector positioning.

---

<sup>9</sup>C.A. Gunn, R. B. Oberer, L. G. Chiang, R. N. Ceo, *A Generalized Finite Source Calibration Factor: A Natural Improvement to the Finite Source Correction Factor for Uranium Holdup Measurements*, Document No. Y/DX-2525, January 2003.

<sup>10</sup>This bias was also observed by Donald L. Sorenson at Fluor Hanford, B. D. Keele, T. L. Welsh, D. K. Balmer, K. D. Bonser, M. Cameron, K. L. Chase, T. D. Cooper, E. W. Curfman, S. T. Hurlbut, J. D. James, J. A. Pestovich, J. Pestovich, Jr. and V. L. Jennings, *Status of Portable Non-Destructive Assay at the Plutonium Finishing Plant*, 46th Annual Meeting of the Institute of Nuclear Materials Management, Phoenix, AZ July 10-14, 2005

For the purposes of the average detector response, the uniformity assumption need only to be statistical. Another purpose of the distribution assumption, however, is for self attenuation. A uniform distribution of uranium bearing material results in a minimum self attenuation. Any deviation from uniformity increases the self attenuation. The assumption therefore results in a negative bias. One way to eliminate the bias is to decrease the width estimate for point and line sources for the purpose of self attenuation. The width for detector response does not have to be equal to the width for self attenuation. Another remedy is needed for area sources.

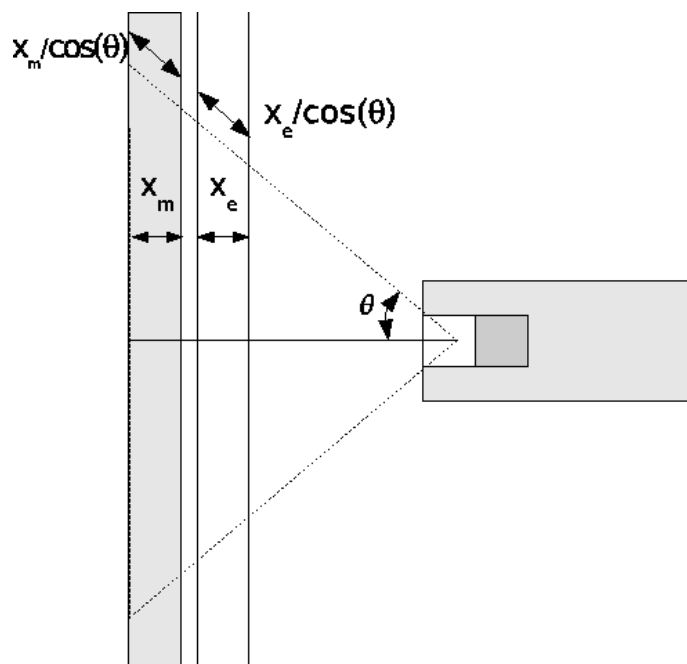


Figure 9: The narrow field-of-view assumption uses  $x$  for attenuation corrections when the apparent path length through the equipment can vary from a minimum of  $x$  to  $x / \cos(\theta)$ .

## 4 Attenuation Corrections

The count rate from the holdup deposit is corrected for intervening material such as equipment or container walls. Typically the correction is  $e^{\mu \rho x}$  where  $x$  is shown in Figure 9. In reality  $x$  is the minimum path length seen by gamma rays. Because a minimum distance is used for the attenuation correction, a negative bias is introduced. Although sometimes called the far-field approximation, this is really a narrow-field-of-view approximation.<sup>11</sup>

<sup>11</sup>R. B. Oberer, C. A. Gunn, L. G. Chiang, *Far-Field Approximation in the Generalized Geometry Holdup (GGH) Model*, Document No. Y/DK-2176

## 5 Background Subtraction

Background from other sources is also detected during a measurement. Typically this background is measured with an unobstructed view of the background. For an area source, the material and equipment obstruct the background. In the traditional GGH method, the background is reduced by the attenuation of the equipment layer for area deposits. The background is not reduced for obstruction by the material nor the equipment for point and line deposits. These simplifications result in an overstatement of background and therefore a negative bias is introduced into the measurement. A more rigorous treatment for background was developed by Y-12.<sup>12</sup> Background should be reduced for attenuation by the uranium bearing material for all geometries and for equipment for point and line geometries.

Table 1: Summary of inherent biases in the Generalized Geometry Model

Source of Bias	Direction of Bias
traditional finite source	negative
uniform distribution	negative
detector jitter	negative
narrow field-of-view	negative
background subtraction	negative

## 6 Conclusion

The GGH holdup model has an inherent negative bias from several sources. Typically the magnitude of the bias is small. However when self attenuation becomes large, the bias can become extreme. Also because biases accumulate, unlike random errors, the total result of a bias can be large. There are relatively easy remedies for all of the biases presented in this paper. The answer is to correct for the biases.

## References

---

<sup>12</sup>R. B. Oberer, C. A. Gunn, L. G. Chiang, *Improved Background Corrections for Uranium Holdup Measurements*, Document No. Y/DK-2107, June 2004.